Assume the entire process to be temperature independent. Then eqs. (61) and (94d) can be combined in the form

$$(95) \qquad dp/dt = a^2 d\varrho/dt + [(m_1 n_2 - m_2 n_1)/(l_1 m_2 - l_2 m_1)](f_{eq} - f)/\tau \,,$$

where m_1, n_1, etc., are as defined in Sect. 5 and a is the frozen sound speed, *i.e.*, $\sqrt{dp/d\varrho}$ with $f = \text{constant}$. Equation (95) is then identical in form to the stress-relaxation equation, eq. (77). Following the procedure described there, we form the characteristic equations, assume the first shock velocity is equal to sound speed, and obtain

$$(96) \qquad dp_1/dt =$$
$$= -[(m_1 n_2 - m_2 n_1)/(l_1 m_2 - l_2 m_1)] \cdot$$
$$\cdot (f_{eq} - f)/2\tau \,,$$

where p_1 is the amplitude of the transition shock. Assuming $\Delta V = V_2 - V_1 = \text{constant}$ as before and $C_{p1} = C_{p2}$, eq. (96) becomes

$$(97) \qquad dp_1/dt = -(\Delta V f_{eq}/2\tau) \, dp_1/dV \,.$$

Fig. 24. – Decay of α-phase wave in iron. (Ref. [9]). Driving pressure $= 200$ kb; $\tau = \frac{1}{3}$ μs; $D_I \tau = 0.17$ cm; • temperature-independent; × temperature-dependent.

Here we have assumed that $f = 0$, *i.e.*, the material is in the metastable first phase at the peak of the first shock. If $dp_1/dV = \text{constant}$, eq. (94$e$) gives

$$(98) \qquad f_{eq} = \begin{cases} 1\,, & V_1 \leqslant V_t) + \Delta V, \\ (V_1 - V_t)/\Delta V = [(p_1 - p_t)/\Delta V] \, dV_1/dp\,, & V_t + \Delta V \leqslant V_1 \leqslant V_t, \\ 0\,, & V_t \leqslant V_1, \end{cases}$$

where V_t, p_t represent the intersection of the phase 1 Hugoniot with the mixed-phase boundary. Combining eqs. (97) and (98) and integrating gives

$$(99a) \qquad p_1 = p_0 - (x \Delta V/2D_1 \tau) \, dp_1/dv\,, \qquad V_1 \leqslant V_t + \Delta V,$$

$$(99b) \qquad = p_t + (p_0 - p_t) \exp[-x/2D_1\tau]\,, \qquad V_t + \Delta V \leqslant V_1 \leqslant V_t,$$
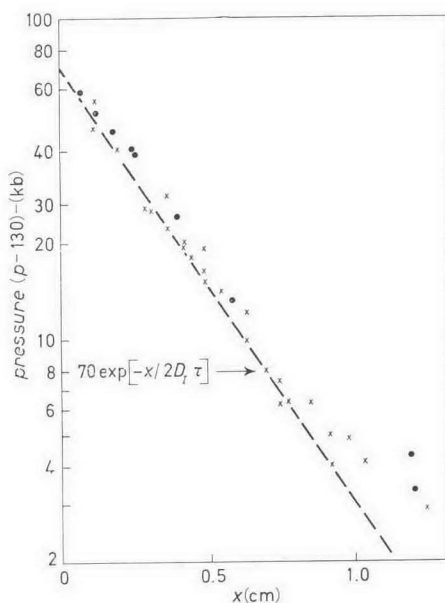
$$(99c) \qquad = p_t\,, \qquad V_1 = V_t,$$

where $x = D_1 t$, D_1 is the first shock velocity, and p_0 is the driving pressure. For $p_0 = 200$ kb, eq. (99b) applies. The result is shown in Fig. 24.

The graphs in Fig. 22-24 were calculated with $\tau = \frac{1}{3}\,\mu s$. In order to determine the relation between rise time in the second shock and τ, the constitutive relations can be combined with eqs. (4)-(6) to determine the steady profile in the second shock. The result of that integration is that the rise time $= 2.25\tau$. Unfortunately the theory of the rates at which solid-solid phase transitions occur is not developed to the state where predictions of experimental results can be made or where experimental results can be used to determine physical parameters. It is evident that careful experiments and creative theoretical work are both required in this area.

8. – Mechanical effects.

The most obvious terminal effects of shock experiments in solids are fractures produced by wave interactions. The geometry of fracture depends principally on the paths of intersecting tensile waves; limiting conditions for fracture are determined by dynamic strength of the material, which may depend on geometry and certainly depends upon stress rate or strain rate.
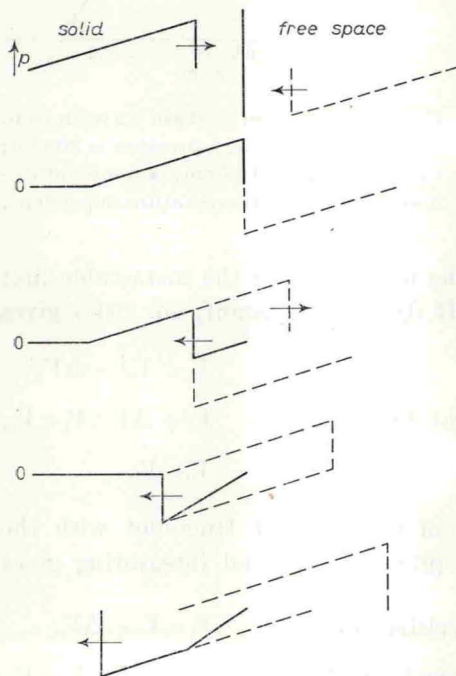

Fig. 25. – Reflection at a free surface, linear waves.

The geometry is indicated in Fig. 25 and 26 for the simplest case, plane spall. In Fig. 25 are shown a sequence of pressure profiles of a compressive pulse approaching and being reflected from a free surface. Propagation and interactions are assumed linear for purposes of illustration. The solid curve is the real pressure profile; the dashed curves represent the pulses from which it is composed. The interaction of the rarefaction in the incident pulse with the reflected rarefaction from the free surface produces tensile stresses in the specimen. If these produce the conditions required for fracture at any point and time, fracture is initiated at that point and time. This is illustrated more precisely in Fig. 26. In 26 a) is shown an (x, t) diagram for the process of Fig. 25. The incident shock, $\mathscr{S}_+$, is